1 Introduction

Hatcher has recently described a logical proof of the existence of God based on three simple postulates. ¹ The proof is based on a similar one first advanced by a twelfth century Muslim philosopher by the name of Avicenna. ² All three of the assumptions put forth by Hatcher are fairly reasonable and straight-forward in the sense that many of us take them for granted in our everyday lives, particularly those working in rational fields, although as we shall see, some of them may need some modification, in which case the proof does not hold. As Hatcher puts it, logic is the science of deriving non-obvious conclusions from obvious assumptions. ³ Hatcher also states that he is interested only in providing a minimal proof of God-that is he is interested in answering the question, "what are the minimal assumptions necessary to prove the existence of God?" without necessarily making any inferences as to what exactly God is like. ⁴

2 The proof

Here we present Hatcher's proof of the existence of God. We begin with a set of definitions. An **object** we will denote by a letter. These we will leave undefined, but presume that such objects correspond to actual phenomena in the real world. The terms object, phenomenon and entity we will use interchangeably in the following discussion. Here we have made our first assumption: that something actually exists, a claim that an Eastern mystic might dispute. ⁵ Further, we will distinguish between singular objects and composite objects and use set theory to relate the two. The latter correspond to sets in set theory and by convention will be denoted by lowercase letters, while the former correspond to members of sets and will be denoted by uppercase letters. The following set notation will be used:

1. $a \in B$ means that a is a **member** of B, that is B is composed of a number of objects, of which a is one

 $^{^1{\}rm This}$ proof is outlined in Hatcher 1998, chapter 3, section 4, pp. 82-86 and appendix II, pp. 139-141. Other versions are also contained in Hatcher 1996 and Hatcher 1990

²Avicenna's proof can be found in Arberry 1951 chapter 25, and Morewedge 1973.

 $^{^{3}}$ This was taken from a seminar at the University of Waterloo (1999) in which Hatcher presented his proof

⁴Hatcher terms this a minimalist approach. From the notes that I took during the seminar: "Minimalism–makes no gratuitous ontological assumptions… Reductionism [is] not the same thing– minimalist as well as absolute assumptions about what is and what isn't. [Minimalism] intends objective not normative discourse." From *Love, Power and Justice*: "Notice that we have used every one of our three principles in deriving this conclusion." (Hatcher 1996 p. 141)

 $^{^5}$ "The only extra-logical assumption we have used is our explicit appeal to the fact that something exists." (Hatcher 1996 p. 141)

2. A \subset B means that A is a **subset** of B such that for every a \in A, a \in B

3. A \cup B (A union B) means to combine all the members of A with all the members of B

4. $A = \{a1, a2, a3, \dots an\}$ curly braces denote membership in a set.

5. If $A \in B$ or $A \subset B$ then we will say that A is a **part** of B.

The notion of causation is fundamental to Hatcher's theory and is also considered to be self-evident.

6. We denote $A \rightarrow B$ as meaning A **causes** B, that is, A exists by virtue of B or B is sufficient for A. One could presumably also say, if A then B. Causation is meant in a "trans-temporal" and very general sense. Logical causes are not excluded and B does not necessarily follow A in time, although "retro-causation" is not explicitly allowed and certain forms of it would conceivably violate the limitation principle (see below).

There are three axioms which Hatcher purports prove the existence of God.

7. Axiom I is the **principle of sufficient reason**. Every object has a cause, whether that object be itself or another object. However, Hatcher contends that an object cannot be both self-caused and other-caused. The claim is that if $A \to B \to A$ then B = A.

8. Axiom II Hatcher terms the **potency principle**. If $A \to B$ then for every b, $A \to b$, where $b \in B$. As a corollary to axiom II:

8a. If $A \to B$ then for every $C, A \to C$ where $C \subset B$.

9. Axiom III Hatcher terms the **limitation principle** which states that if $b \in A$ then A cannot cause b. This seems fairly common sense-how can a system cause one of its own parts unless that part is already extant to complete the system? The example of the body producing a new cell does not counter this argument as the body is now a new system. As a corollary:

9a. If $B \rightarrow A$, then A cannot cause B.

We are now ready to prove the existence of "God", which we define as a unique, singular, uncaused (or self-caused) universal cause. It is easy to see that under Hatcher's three axioms, no composite object can be self-caused as this contradicts the limitation principle: if $A \to A$ then for every $a \in A, A \to a$. If we define V to be a composite object comprising all of reality, past, present and future, then we can ask the question, "What causes V?" The only thing which satisfies all of the above three axioms is the object defined in the first sentence of this paragraph, what we traditionally think of as God.

10. $G \rightarrow G \rightarrow V$ where $G \in V$ and G is unique.

There seems to be a problem with Hatcher's notion of causation as he presented in a seminar and in his Hatcher's book, *Love Power and Justice*, upon both of which this proof is based. ⁶ This is actually quite easy to fix. First, we note that with the current notion of causation, we can say without contradicting the definition of cause that $V \rightarrow G$, in direct contradiction with the third axiom. The two most obvious ways of fixing this are not adequate. We can say that a cause must be minimal, but in a choice of two possible causes, one of which has more components than the other, we have limited ourselves to only one of two perfectly admissible causes. Second, we can say that a cause must be necessary as well as sufficient, however then our notion of causation is incomplete. The following restrictions on cause will fix the problem:

11. If $A \to B$ then for every $a \in A$, A - a does not cause B.

Note we have used the notion of set subtraction here which we proceed to define:

12. C = A - B: C is the difference of set A and B such that for every $b \in A$ and $b \in B$, then $b \in C$.

3 A counter-example

Suppose V is piecewise caused, i.e.:

⁶Hatcher has the following to say about causation or causality: "A *phenomenon* A is some portion of reality, and causality is a relationship between two phenomena A and B, which holds whenever A is a cause of B (symbolized $A \rightarrow B$). This means that A contains *sufficient* reason for the existence of B." (Hatcher 1996 p. 83) (Italics are Hatcher's.) From my seminar notes: "A \rightarrow B, 'B exists by virtue of A'... [This is a] minimal not maximal truth about causality."

14. a, b, c, ... , $n \in V$

15. $V \equiv A \cup B \cup C \cup \dots \cup N \cup \{a, b, c, \dots n\}$

What causes V?

16. { a, b, c, ... n } \rightarrow V,

but this does not violate the limitation principle, according to the above.

Actually, under Hatcher's system, this is not strictly valid, but taking it as an example, we begin to see what's wrong with his proof. We have here an example of a composite object which is also self-caused, which we have already stated is invalid under Hatcher's system. However, this I believe to be the weakness of Hatcher's proof. If we have a composite object composed solely of self-caused objects, can we not say that the whole thing is also self-caused? Under Hatcher's logic, the aggregate object forms an entirely new entity which now needs an external cause to get it going, which seems absurd. So I propose to revise Hatcher's system as follows: like objects, a cause can be either singular or composite, where a composite cause can be broken up into two or more disjoint causes. The above is an example of a composite causes. One should use two different symbols for singular versus composite causes:

17. A \rightarrow B such that a cannot cause b where a \in A and b \in B.

18. In the example above, $\{a, b, c, \dots n\} \Rightarrow V$ As soon as we do this we should now limit Axiom III only to singular causes.

4 Critical commentary

Hatcher's proof seems to be for the most part logically consistent. The chief criticism that can be leveled against the logic is that it is essentially tautological. It seems to be deliberately formulated to preclude phenomena that we have described above in the supposed counter-example and to prove that there is a single universal self-caused cause. Hatcher does not deny this-in fact he expressly states it. ⁷ By contrast, we have gone the other way and assumed that composite objects composed of self-caused components can exist. The preceding discussion calls considerable attention to the role of composite objects. Can we have a composite object which is strictly mentally assembled, namely that we have chosen to arbitrarily group a certain set of objects together? In Hatcher's proof, a group of objects which are composite by their very nature must have a common cause. As

⁷As per our initial discussion and note 4. This is also implied in *Love, Power and Justice*: "Let us begin with a clear statement of what we are trying to prove." (Hatcher 1996 p. 139)

soon as we define a set of objects as being composite we have assumed that they have a cause. So as soon as we define V as all of reality, we have already assumed the existence of God. Another puzzling feature of Hatcher's proof is the following:

19. Suppose $a_1 \rightarrow b_1 \rightarrow a_2 \rightarrow b_2, a_3 \rightarrow b_3, \dots a_n \rightarrow b_n$

20. If A={ $a_1, a_2, a_3, \dots a_n$ } and B={ $b_1, b_2, b_3, \dots b_n$ } then we can say that A \rightarrow B without violating any of the axioms

21. But not, paradoxically if $A \subset B$

While the logic of Hatcher's proof may be essentially consistent, in order for it to have any value, it ought to have some correspondence to the real world. Presumably the objects to which he refers in his proof are not just mental constructs but must also correspond to actual objects in the real world. Whether or not a mental construct is also an object in our system is a point of contention. We can level several criticisms at both the principle of sufficient reason, and at his definitions. We begin with the definitions. Hatcher assumes that there are such a thing as indivisible objects, a claim that is easy to call into question. I would argue that the divisions between objects are to some extent arbitrarily defined and boundaries are simply where we choose to make them. Some objects are more obviously separate than others. For something like a rock, it is very easy to define boundaries while for things like sub-atomic particles and more abstract entities like ecosystems, it is not so easy to define. To some degree Hatcher has resolved this difficulty by limiting a composite entity to being grouped by a common cause as discussed above. We can then ask the question, however, does it make sense to define all of reality as a single composite entity?

We can also call into question the notion of causation. For classical systems causation is typically continuous whereas in the proof it is assumed to be discrete. In quantum systems causation is even more elusive, but in neither case does the notion of causation form an explicit component of the logic. The most typical example of a questionable causal relationship in quantum mechanics is that of nuclear decay. There is always an indeterminate time between now and when an unstable nucleus decays. Can anything be said to have "caused" the decay of the nucleus? We could argue that the instability of the nucleus caused the decay, but that still leaves the indeterminate time span-what causes that specific decay time rather than any other? Proponents of "neo-classicism" will argue for the hidden variable theory-something has caused the decay, we just don't know what it is.

We are attempting to strike directly at the heart of the foundations of rationality as it is normally conceived, which is the principle of sufficient reason. The normal convention is to assume that everything has a cause or an explanation. Unfortunately if we do that then we are left with two alternatives: either an infinite regress of causes or a circular chain. Aristotle argues that either alternative is absurd and so there must be an uncaused cause. ⁸ Of course as soon as we admit one we are forced to admit that there may be others! Hatcher allows for uncaused (or self-caused) causes, but excludes the possibility that an object may be both. This is to prevent the aforementioned circular chain. Again, I can think of a good practical example to suggest that this is not the case. In the case of a cellular automaton we can have a set of stationary, periodic states-the last one in the sequence produces the first one in a circular repetition. Clearly each state is distinct, although this example is slightly flawed as a cellular automaton has to be started at some point and will most likely not run on forever! However we can imagine similar natural systems which were not started at any point. As a counter argument, perhaps each repetition of a given state is only the same conceptually, while in actuality we have an entirely new state. For a true "circular chain" of causation, we actually need some form of retro-causation, but as mentioned previously, this is not explicitly excluded.

Finally we can ask if the notion of V defined as all of reality makes any sense. Can we arbitrarily define an object composed of all of reality and talk about it as if it were somehow a separate entity? In this case we are putting ourselves in the position of "objective observer" as if we were somehow standing apart from reality but by definition we cannot be. Not even God occupies such a privileged position according to Hatcher! I think we should be extra cautious whenever drawing up such broad and simplistic models of reality and by extension of all rational systems. It is an interesting curiosity, and while my own bias is towards theism, I would argue that the proof is of relatively little merit either in telling us anything about God or about reality.

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⁸Aristotle's proof of God can be found in his Metaphysics Book α , chapter 2. A version of it is also contained in the *Summa Theologica* of Thomas Aquinas, Question II.

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